Numerical Physics: Synchrotron radiation effect on dynamical vacuum with localized photon stops in a VLHC Stage 2 beam screen section - Steady state

■ Abstract

The objective of this paper is to the discuss the numerical approach to the calculation of dynamical vacuum when synchrotron radiation hits a photon stop arranged in a 1m beam screen section, of the Very Large Hadron Collider VLHC Stage 2.

■ Photodesorption from a photon stop induced by synchrotron radiation

The diffusion equation describing the flow of a non uniform gas is given by the Fick's second law

$$\frac{\partial n(\mathbf{z}, t)}{\partial t} = \nabla \cdot (D \; \nabla \cdot n(\mathbf{z}, t)) \qquad (1)$$

where D is the diffusion coefficient, and $n(z,t)$ represents the gas density. We assume that in the VLHC Stage 2, the synchrotron radiation induces photodesorption by hitting a localized photon stop arranged in a beam screen 1 *m* long section. Furthermore, ion pumps are located at 5 *m* from the photon stop, as shown in Fig. 1. Holes in the beam screen evacuate the photodesorbed gas from the beam pipe. Cryopumping is supplied by a coaxial 4^oK cold surface. We need to generalize the Fick's equation to include cases when a source of gas is present and a distributed pumping is provided. In the beam screen section

$$A_c \frac{\partial n(\mathbf{z}, t)}{\partial t} = \nabla \cdot (D A_c \; \nabla \cdot n(\mathbf{z}, t)) - S' \; n(\mathbf{z}, t) + Q \; \delta(z) \qquad (2)$$

where A_c is the beam pipe cross section, S' the distributed pumping speed in the beam screen, Q the photodesorbed gas load, and the delta function $\delta(z)$ represents the gas source localized at the photon stop. The photodesorbed gas load is proportional to the photon flux generated in a dipole magnet $Q = l_d \, \dot{\Gamma} \, \eta_o$ where l_d is the magnet length, $\dot{\Gamma}$ is the total photon flux, and $\eta_o$is the photodesorption yield.

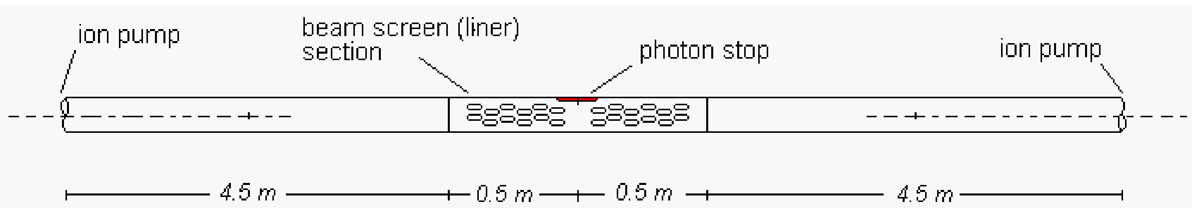

Fig. 1. The photon stop is arranged inside the beam pipe in a limited beam screen 1 *m* long section. Ion pumps are located at 5 *m* from the photon stop.

■ Steady State case

We consider here the steady state problem $\partial n(\mathbf{x}, t)/\partial t \simeq 0$, with D and A_c uniform in space. Eq. (2), along the beam pipe axial length z, reads

$$A_c D \frac{\partial^2 n(z, t)}{\partial z^2} - S' n(\mathbf{z}, t) + Q \,\delta(z) = 0 \qquad (3)$$

where $S' \neq 0$ in the beam screen section, and $S'=0$ in the 4.5 m sections ending with ion pumps. Boundary conditions apply at the ion pump locations z=±5 m

$$\mp D A_c \frac{\partial n(z)}{\partial z}\Big|_{z=\pm z_2} = S\, n(\pm z_2) \qquad (4)$$

In particular, solving eq. (3) for an infinitely long beam screen section, with an axially uniform pumping speed, the *analytic solution* results

$$n(z) = \frac{Q}{2\sqrt{AS'D}} e^{-\sqrt{\frac{S'}{AD}}\, z}\, \theta[z] + \frac{(1-\theta[z])}{2\sqrt{AS'D}} Q e^{\sqrt{\frac{S'}{AD}}\, z} \qquad (5)$$

$\theta[z]$ is the Heaviside step function.

■ Numerical approach. Combined sections: beam screen and ion pump section.

We solve eq. (3) by a *finite element* numerical approach, for the combined sections shown in Fig. 1. Where, the beam screen section is defined by $S' \neq 0$ for $-0.5m \leq z \leq 0.5m$, and $S' = 0$ elsewhere. In this approach the density is defined on a lattice of discrete z - values $n_i = n(i\,\Delta z)$. To discretize eq. (3) the differential quotients have to be replaced by difference quotients

$$\frac{\partial^2 n}{\partial z^2} \to \frac{1}{\Delta z^2}(n_{i-1} - 2 n_i + n_{i+1})$$

$$\frac{\partial n}{\partial z} \to \frac{1}{2\Delta z}(n_{i+1} - n_{i-1})$$

the continuous Dirac function $\delta(z) = \frac{1}{\pi} \frac{k}{1+k^2 z^2}$, with $k \to \infty$, can be discretized as

$$\delta(z_i) \to \frac{1}{\pi} \frac{k}{1 + k^2 (i\,\Delta z)^2} \qquad (6)$$

where k should be properly normalized to satisfy the condition $\sum_{i=-\infty}^{\infty} \delta(z_i)\,\Delta z = 1$, equivalent to $\int_{-\infty}^{\infty} \delta(z)\, dz = 1$. Thus, with the substitution $\alpha = AD/\Delta z^2$, eq. (3) becomes a set of difference equations of the form

$$-\alpha\, n_{i-1} + (S' + 2\,\alpha)\, n_i - \alpha\, n_{i+1} \;=\; \frac{k}{\pi\,(1 + i^2\, k^2\, \Delta z^2)}\; Q\,. \qquad (7)$$

The 10 *m* long section is divided in N intervals, or N+1 nodes. The ion pump locations $z=\pm z_2$ correspond to i = 0 and i = N. For i=1 and i=N, $S'=0$ and $S\neq 0$, and the boundary conditions (4) at $z= - z_2$ and $z= + z_2$ reads respectively

$$n_0 \;=\; -2\,\Delta z\left(\frac{S}{A D}\right) n_1 + n_2\;, \qquad n_N \;=\; -2\,\Delta z\left(\frac{S}{A D}\right) n_{N-1} + n_{N-2} \qquad (8)$$

where *S* is the ion pumping speed. Substituting eqs. (8) in eq. (7) we obtain the boundary conditions

$$\begin{aligned}
\left(2\,\alpha + \frac{2\,S}{\Delta z}\right)\, n_1 - 2\,\alpha\, n_2 \;&=\; \frac{k\,Q}{\pi\,(1 + k^2\,\Delta z^2)}, \\
\left(2\,\alpha + \frac{2\,S}{\Delta z}\right)\, n_{N-1} - 2\,\alpha\, n_{N-2} \;&=\; \frac{k\,Q}{\pi\,(1 + k^2\,(N-1)^2\,\Delta z^2)}
\end{aligned} \qquad (9)$$

Defining $\vec{n} = \{n_0, n_1, \ldots, (n_{N-1}\})^T$ and $\vec{\delta} = \{\delta(z_1), \delta(z_2), \ldots, (\delta(z_{N-1})\})^T$, the above equation (7) can be written in a matrix-vector form as

$$M\;\vec{n} \;=\; \vec{\delta} \qquad \text{or} \qquad \mathscr{L} = M^{-1}, \qquad \vec{n} \;=\; \mathscr{L}\;\vec{\delta}$$

with the matrix elements, of the resulting $(N-1)\times(N-1)$ tridiagonal matrix, given as follows:

$$M_{i,j} \;=\; \left(S' + 2\,\frac{A D}{\Delta z^2}\right), \qquad \textit{for } i = j\,, \ \textit{with } S' \neq 0 \textit{ only in the beam screen section}$$

$$M_{i,j} \;=\; -\frac{A D}{\Delta z^2}\;, \qquad \textit{for } i = j+1$$

$$M_{i,j} \;=\; -\frac{A D}{\Delta z^2}\;, \qquad \textit{for } i = j-1$$

$S'\neq 0$ only for $\frac{N}{2} - \frac{0.5\,m}{\Delta z} \;\leq\; i \;\leq\; \frac{N}{2} + \frac{0.5\,m}{\Delta z}$, corresponding to the beam screen section (central region of the matrix), while S'=0 elsewhere. Furthermore, the boundary conditions (9) results in the matrix elements

$$M_{1,1} \;=\; \left(2\,\frac{A D}{\Delta z^2} + 2\,\frac{S}{\Delta z}\right), \qquad M_{1,2} \;=\; -2\,\frac{A D}{\Delta z^2}$$

$$M_{N-2,N-1} \;=\; -2\,\frac{A D}{\Delta z^2}\;, \qquad M_{N-1,N-1} \;=\left(2\,\frac{A D}{\Delta z^2} + 2\,\frac{S}{\Delta z}\right)$$

and $$M_{i,j} \;=\; 0\;, \qquad \textit{for } |i-j| > 1$$

Finally, the density profile is obtained by $\vec{n} = \mathcal{L}\,\vec{\delta}$.

The density profile computed numerically is shown in Fig. 2, considering a CO gas and assuming the following parameters (see also [1]): distributed pumping speed in the beam screen section *S'=60 l/s-m*, total photon flux $\dot{\Gamma}$= 1.2 10^{16}*ph/s-m*, CO photodesorption yield for copper η_o= 1.5 10^{-3} *mol/ph* [5], dipole magnet length l_d =14 *m*, gas load $Q = l_d\,\dot{\Gamma}\,\eta_o$ = 2.5 10^{14}*molec/sec*, pumping speed of the ion pumps *S*=30 *l/s*, average beam pipe radius *a*=0.0125*m*, A_c=4.9 10^{-4} m^2, *D* =10.22 m^2/s. With numerical parameter: N=100, Δz=0.1 *m*.

Furthermore, the numerical solution has been checked with the analytical solution (5) as shown in Fig. 3. In particular, the analytic solution (5) is valid for an infinitely long beam screen section, where the density $n(\pm\infty)\to 0$. Thus, in the numerical calculation we consider $S'\neq 0$ for all the elements of the matrix, and impose the boundary conditions $n(\pm z_2) = 0$, with $z_2 \gg 0$.

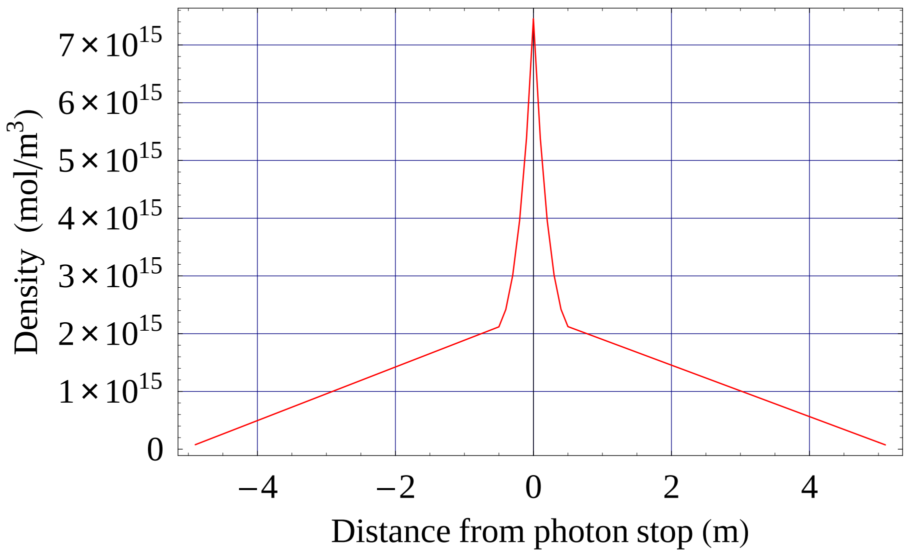

Fig. 2. CO density profile as a function of the distance from the photon stop, in a 10*m* long combined beam screen and ion pumps section, with *S'=60 l/s-m* and *S*=30 *l/s*.

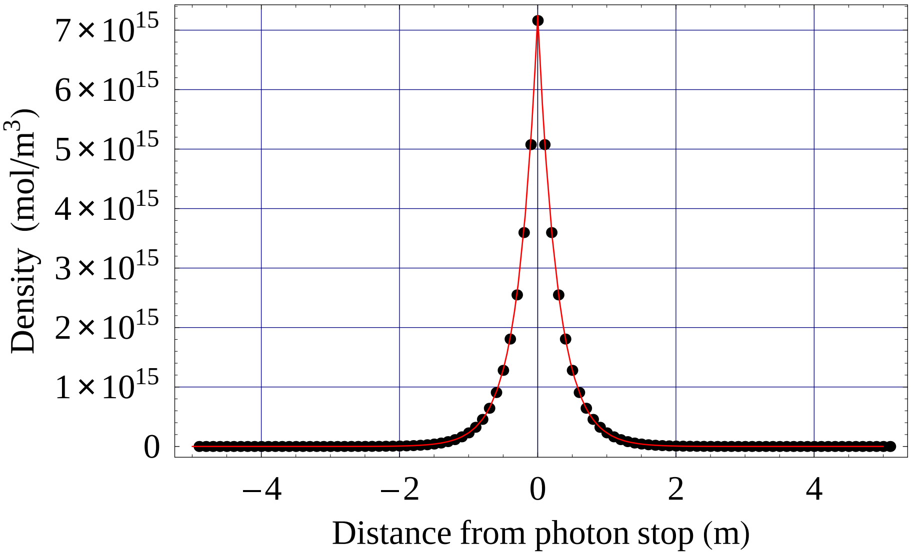

Fig. 3. Computed numerical density profile (dots) compared with the analytical solution eq. (5), for an infinitely long beam screen section.

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Numerical Physics: Synchrotron radiation effect on dynamical vacuum with localized photon stops in a VLHC Stage 2 beam screen section - Time dependent problem

■ Abstract

The objective of this paper is to the discuss the numerical approach to the calculation of dynamical vacuum when synchrotron radiation hits a photon stop arranged in a 1m beam screen section, of the Very Large Hadron Collider VLHC Stage 2.

■ Photodesorption from a photon stop induced by synchrotron radiation

The diffusion equation describing the flow of a non uniform gas is given by the Fick's second law

$$\frac{\partial n(\mathbf{z},t)}{\partial t} = \nabla \cdot (D\ \nabla \cdot n(\mathbf{z},t)) \qquad (1)$$

where D is the diffusion coefficient, and $n(\mathrm{z},t)$ represents the gas density. We assume that in the VLHC Stage 2, the synchrotron radiation induces photodesorption by hitting a localized photon stop arranged in a beam screen 1 *m* long section. Furthermore, ion pumps are located at 5 *m* from the photon stop, as shown in Fig. 1. Holes in the beam screen evacuate the photodesorbed gas from the beam pipe. Cryopumping is supplied by a coaxial 4^oK cold surface. We need to generalize the Fick's equation to include cases when a source of gas is present and a distributed pumping is provided. In the beam screen section

$$A_c \frac{\partial n(\mathbf{z},t)}{\partial t} = \nabla \cdot (D A_c\ \nabla \cdot n(\mathbf{z},t)) - S'\ n(\mathbf{z},t) + Q\ \delta(\mathrm{z}) \qquad (2)$$

where A_c is the beam pipe cross section, S' the distributed pumping speed in the beam screen, Q the photodesorbed gas load, and the delta function $\delta(\mathrm{z})$ represents the gas source localized at the photon stop.

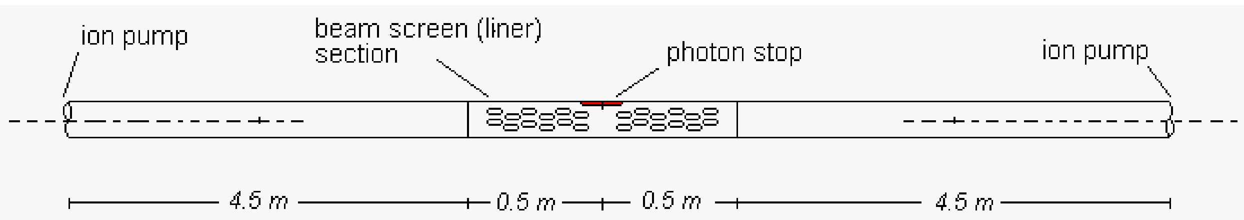


Fig. 1. The photon stop is arranged inside the beam pipe in a limited beam screen 1 *m* long section. Ion pumps are located at 5 *m* from the photon stop.

■ Time dependent problem

We consider here the time dependent problem $\partial n(\mathbf{x}, t) / \partial t \neq 0$, with D and A_c uniform in space. Eq. (2), along the beam pipe axial length z, reads

$$A_c \frac{\partial n(\mathbf{z}, t)}{\partial t} = A_c D \frac{\partial^2 n(z, t)}{\partial z^2} - S' \, n(\mathbf{z}, t) + Q \, \delta(z) \qquad (3)$$

where $S' \neq 0$ in the beam screen section, and $S'=0$ for the 4.5 *m* sections ending with ion pumps.

In the general problem we condsider a representative example of SR generated by the passage of a single bunch in a 14 *m* long dipole magnet. Then, we compute the density profile of the photodesorbed gas molecules generated at t=0 at the photon stop location, and its evolution in time. We assume that all the emitted radiation is intercepted by a photon stop [2]. The density at the photon stop location is given by

$$n(z, 0) = \dot{\Gamma} \; l_d \, \tau_d \; \eta_0 \; \delta(z) \qquad (4)$$

where $\dot{\Gamma}$ is the total photon flux, l_d=14 m and τ_d are respectively the magnet lenght and the time spent by a single bunch in the magnet, and η_0 is the photodesorption yield. Boundary conditions apply at the ion pump location

$$\mp D A_c \frac{\partial n(z, t)}{\partial z} \bigg|_{z=\pm z_2} = S \, n(\pm z_2) \qquad (5)$$

where z_2=5*m* is the distance of the ion pump from the photon stop, and S is the ion pumping speed.

■ Numerical approach.

We solve eq. (3), by a *finite element* numerical approach, for the combined sections shown in Fig. 1. In this approach the density is defined on a lattice of discrete (z,t) - values

$$n_i^{(j)} = n \, (i \, \Delta z, j \, \Delta t),$$

$$\vec{n}^{(j)} = \{n_1^j, n_1^j, \ldots, n_{N-1}^j\}$$

We will show that the density profile $\vec{n}^{(j)}$ again follows a sort of exponential decay and that the eigenvalues and eigenfunctions of the original calculus can be approximated by the eigenvalues and functions of that matrix.

$$\vec{n}^{(j)} = \mathcal{L} \; \vec{n}^{(j-1)} \,, \qquad \vec{n}^{(j)} = \mathcal{L}^j \; \vec{n}^{(j-1)}$$

$\vec{n}^{(0)}$ ... initial profile at time $t=0$!

we will use indifferently the notation $n_i^{(j)}$ or n_i^j.

■ Discussion of the problem. Combined sections: beam screen and ion pump section.

The differential equation of the problem is given by (3) with initial and boundary conditions (4-5). To discretize eq. (3), the differential quotients have to be replaced by difference quotients

$$\frac{\partial^2 n}{\partial z^2} \to \frac{1}{\Delta z^2}\,(n^j_{i-1} - 2\,n^j_i + n^j_{i+1})$$

$$\frac{\partial n}{\partial z} \to \frac{1}{2\,\Delta z}\,(n^j_{i+1} - n^j_{i-1})$$

$$\frac{\partial n}{\partial t} \to \frac{1}{\Delta t}\left(n^j_i - n^{j-1}_i\right)$$

$$n \to n^j_i$$

the continuous Dirac function $\delta(z) = \frac{1}{\pi}\,\frac{k}{1+k^2\,z^2}$, with $k\to\infty$, can be discretized

$$\delta\,(z_i) \to \frac{1}{\pi}\,\frac{k}{1+k^2\,(i\,\Delta z)^2}$$

where k should be properly normalized to satisfy the condition $\sum_{i=-\infty}^{\infty}\delta(z_i)\,\Delta z = 1$, equivalent to $\int_{-\infty}^{\infty}\delta(z)\,dz = 1$. The Dirac function term in eq. (3) will be here included in the initial condition (4). Thus, we drop the term including the Dirac function from eq. (3). Eq. (3) becomes a set of difference equations (multiplied by -Δt) of the form

$$-\alpha\;n^j_{i-1} + \left(1 + 2\,\alpha + \frac{\Delta t\,S'}{A}\right)\,n^j_i - \alpha\;n^j_{i+1} = n^{j-1}_i \qquad (6)$$

with the substitution α =D Δt / Δz^2.

Stability considerations: The fully implicit scheme (or backward time), considered here, is unconditionable stable for any choice of the step size Δt [3], contrary to the Forward Time Centered Space (FTCS) scheme. On the other hand, to preserve the accuracy in the small-scale evolution of the solution we choose a step size value Δt small enough to satisfy the condition

$$\frac{2\,D\,\Delta t}{\Delta z^2} = 2\,\alpha \le 1$$

which is the stability criterion for the FTCS approach ▲.

The 10 *m* long section is divided in N intervals, or N+1 nodes. The ion pump locations $z=\pm z_2$ correspond to i = 0 and i = N. For i=1 and i=N, $S'=0$ and the ion pumping speed $S\neq 0$, and the boundary conditions (5) at $z= -\,z_2$ and $z= +\,z_2$ reads respectively

$$n_0^j = -2\,\Delta z\left(\frac{S}{A D}\right) n_1^j + n_2^j\ , \qquad n_N^j = -2\,\Delta z\left(\frac{S}{A D}\right) n_{N-1}^j + n_{N-2}^j\ . \qquad (7)$$

Substituting eqs (7) in eq. (6) we obtain the boundary conditions

$$\left(1+2\ \alpha+\frac{2\,\Delta t\,S}{A\,\Delta z}\right) n_1^j - 2\ \alpha\ \ n_2^j = n_1^{j-1}\ , \qquad \left(1+2\ \alpha+\frac{2\,\Delta t\,S}{A\,\Delta z}\right) n_N^j - 2\ \alpha\ n_{N-1}^j = n_N^{j-1} \qquad (8)$$

The above equations (6,7,8) can be written in a matrix-vector form as

$$M\ \ \vec{n}^{(j)}\ =\ \vec{n}^{(j-1)} \qquad or \qquad \mathcal{L} = M^{-1}, \qquad \vec{n}^{(j)}\ =\ \mathcal{L}\ \vec{n}^{(j-1)}$$

with the matrix elements, of the resulting $(\mathrm{N}-1)\times(\mathrm{N}-1)$ tridiagonal matrix, given as follows:

$$M_{i,j}\ =\left(1+\frac{2\,D\,\Delta t}{\Delta z^2}+\frac{\Delta t\,S'}{A}\right), \qquad for\ \ i=j\ ,\ \ with\ S'\neq 0\ only\ in\ the\ beam\ screen\ section$$

$$M_{i,j}\ =-\frac{D\,\Delta t}{\Delta z^2}\ , \qquad for\ i=j+1$$

$$M_{i,j}\ =-\frac{D\,\Delta t}{\Delta z^2}\ , \qquad for\ i=j-1$$

$S'\neq 0$ only for $\frac{N}{2}-\frac{0.5\,m}{\Delta z}\ \leq\ i\ \leq\ \frac{\mathrm{N}}{2}+\frac{0.5\,m}{\Delta z}$, corresponding to the beam screen section (central region of the matrix), while S'=0 elsewhere. Furthermore, the boundary conditions (8) results in the matrix elements

$$M_{1,1}\ =\left(1+\frac{2\,D\,\Delta t}{\Delta z^2}+\frac{2\,\Delta t\,S}{A\,\Delta z}\right), \qquad M_{1,2}\ =\ -2\,\frac{D\,\Delta t}{\Delta z^2}$$

$$M_{N-2,N-1}\ =-2\,\frac{D\,\Delta t}{\Delta z^2}\ , \qquad M_{N-1,N-1}\ =\left(1+\frac{2\,D\,\Delta t}{\Delta z^2}+\frac{2\,\Delta t\,S}{A\,\Delta z}\right)$$

$$and \qquad M_{i,j}\ =\ 0\ \ , \qquad for\ \ |i-j|\ >1$$

The initial condition is given by substituting the discretized $\delta(z_i)$ function in eq. (4), and reads

$$n_i^0 = \dot{\Gamma}\ \ l_d\ \tau_d\ \eta_0\ \left(\frac{1}{\pi}\ \frac{k}{1+k^2\ (i\,\Delta z)^2}\right) \qquad (4\,')$$

Finally, the density profile at time $t=j\,\Delta\,t$, is obtained by iteration $\vec{n}^{(j)}=\ \mathcal{L}\ \vec{n}^{(j-1)}$.

An example of the results of the numerical computation, is shown in Fig. 2 and 3, assuming the following parameters: distributed pumping speed in the beam screen section *S'=60 l/s-m*, total photon flux $\dot{\Gamma}$= 1.2 10^{16}*ph/s-m*, CO photodesorption yield for copper η_o = 1.5 10^{-3} *molec/ph*[5], dipole magnet length l_d =14 *m*, average beam pipe radius *a*=0.0125 *m*, A_c=4.9 10^{-4} m^2, D =10.22 m^2/s (see also [1]). With numerical parameter: N=100, Δt=50 μs, Δz=0.1 *m.*

▪ The eigenvalue problem

Let λ_i and ψ_i be respectively the eigenvalues and eigenvectors of the matrix $\mathcal{L}$. The first eigenvectors are shown in Fig. 4. The ψ_i should be orthogonal with respect to the metrics of the *linear* problem, see Fig. 5.

$$\langle \psi_i , \psi_j \rangle = \delta_{i,j} \quad , \qquad \langle A_i , B_j \rangle = \sum_{k=1}^{N} A_{i,k} \; B_{k,j} = \text{A B} \qquad (9)$$

The initial condition $\vec{n}^{(0)}$ can be written as a linear combination of the eigenvectors

$$\vec{n}^{(0)} = \sum_i \alpha_i \, \psi_j \quad , \qquad \alpha_i = \langle \psi_j , \vec{n}^{(0)} \rangle$$

the coefficient α_i are shown in Fig. 6. The vector of the j-th timestep is then given by

$$\vec{n}^{(j)} = \Upsilon \, \Lambda^j \vec{\alpha} \quad , \qquad (10)$$

where again, Λ is the eigenvalues diagonal matrix $\Lambda_{i,j} = \lambda_i$ and Υ is the matrix of the eigenvectors ψ_i. The computed solution of the eigenvalue problem (10) is shown in Fig. 7. It is in very good agreement with the numerical solution shown in Fig. 2.

▪ Green function for an infinitely long beam screen section

The numerical calculation has been checked with the Green function solution obtained with the Fourier analysis of the problem. We will give the analytic solution for the time dependent spatial distribution of the density for an infinitely long beam screen tube satisfying the condition that N molecules are puffed into the tube at time t=0 and at the photon stop location z=0

$$A_c \frac{\partial n(\mathbf{z}, t)}{\partial t} = A_c D \frac{\partial^2 n(z, t)}{\partial z^2} - S' \; n(\mathbf{z}, t) + N \; \delta(z) \, \delta(t) \qquad (11)$$

Using the Fourier series representations of the Delta function

$$\delta(z) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{ikz} \, dk \qquad \delta(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-i\omega t} \, d\omega$$

and considering that the density *n(z,t)* is related to its Fourier transform $\tilde{n}(z,t)$ as

$$n(z, t) = \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} dk \, d\omega \;\; e^{i(kz-\omega t)} \; \tilde{n}(z, t) \qquad (12)$$

substituting the previous representations in eq. (11) we obtain the expression for $\tilde{n}(z,t)$

$$\tilde{n}(z,t) = \frac{N}{A_c} \frac{i}{\omega + i D k^2 + i \frac{S'}{A_c}}$$

the solution is obtained from eq. (12) and is given by

$$n(z,t) = \frac{1}{(2\pi)^2} \frac{N}{A_c} \int_{-\infty}^{\infty} dk\, e^{ikz} \int_{-\infty}^{\infty} d\omega\; e^{-i\omega t} \frac{i}{\omega + i D k^2 + i \frac{S'}{A_c}}$$

following [4], we obtain the density profile for an infinitely long beam screen section

$$n(z,t) = \frac{N}{A_c} \frac{e^{-\frac{z^2}{Dt}}}{\sqrt{4\pi D t}}\, e^{-\frac{S'}{A_c} t} \qquad (13)$$

which is a Gaussian distribution in z with a standard deviation increasing with the square root of time $\sigma=\sqrt{2Dt}$, and with an exponential decay in time due to the beam screen pumping speed.

The solution of the numerical calculation has been compared with the Green function solution (13) as shown in Fig. 8, t=5ms after the synchrotron radiation hits the photon stop. In particular, the Green function solution (13) is valid for an infinitely long beam screen section, where the density $n(\pm\infty)\rightarrow 0$. Thus, in the numerical calculation we consider $S'\neq 0$ for all the elements of the matrix, and impose the boundary conditions $n(\pm z_2) = 0$, with $z_2 >> 0$.

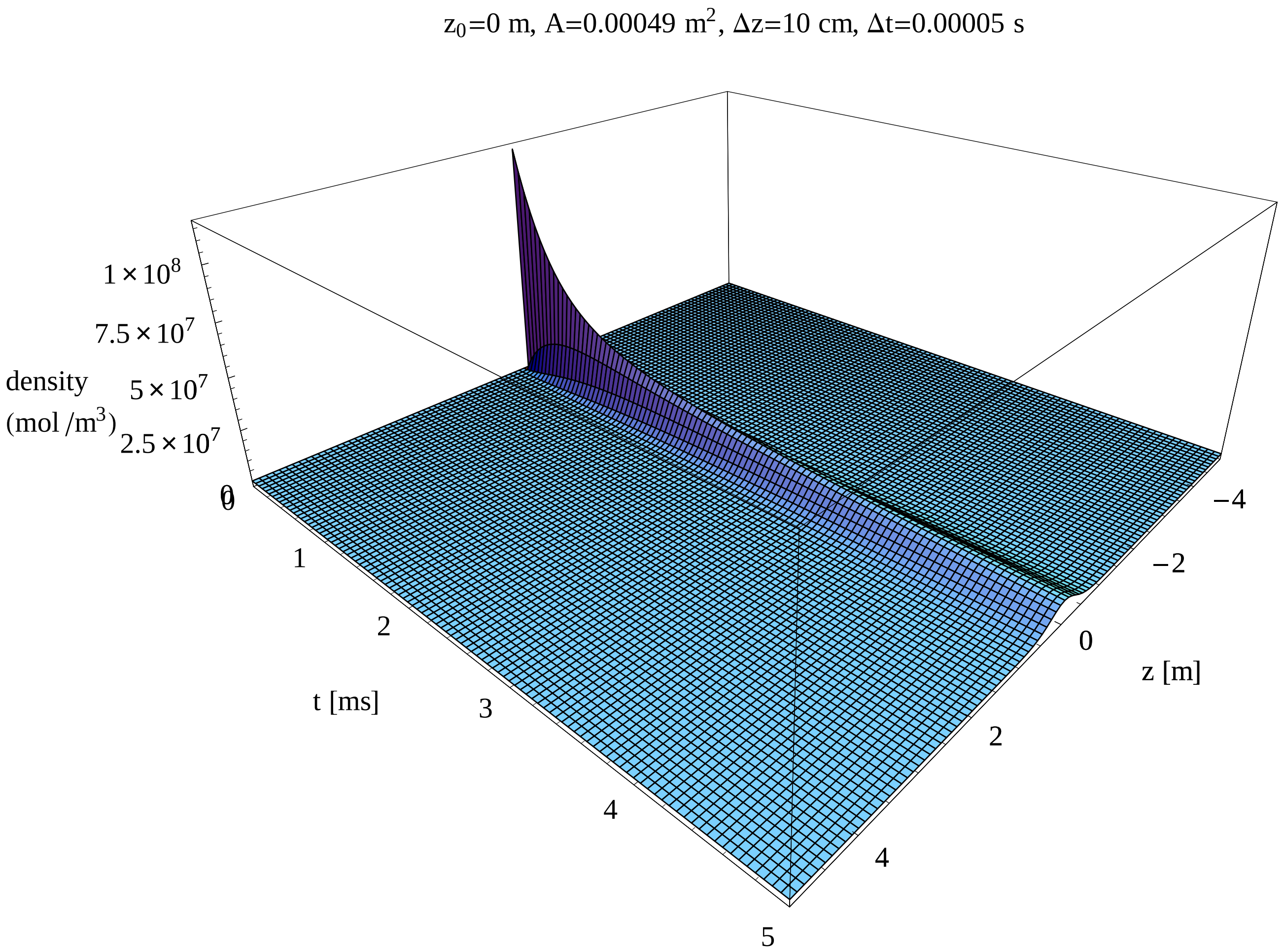
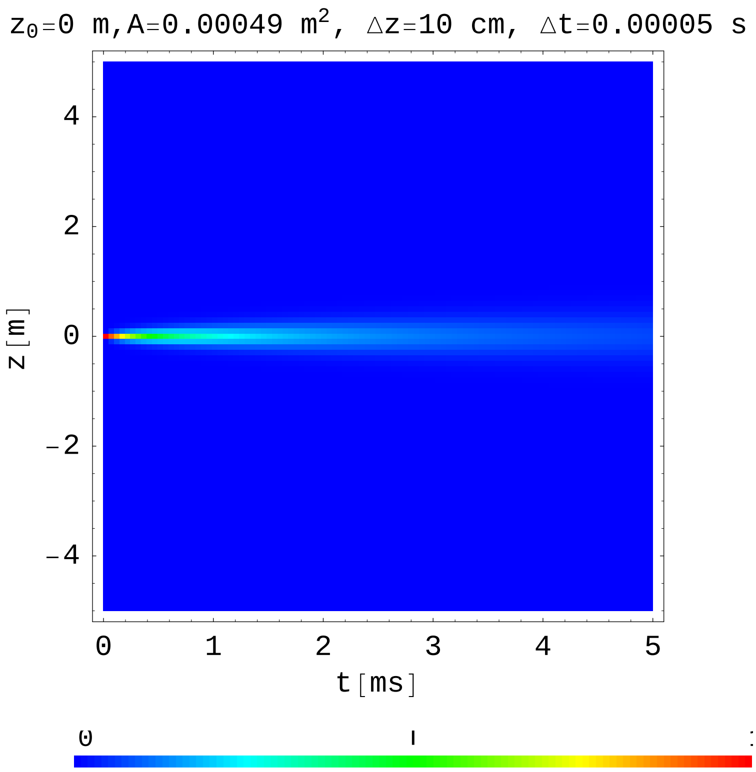


Fig. 2. Density profile in a VLHC Stage 2 combined section, beam screen and ion pump sections. The synchrotron radiation hits the photon stop at t=0. The photon stop located at z=0, is arranged in a 1*m* long beam screen section.



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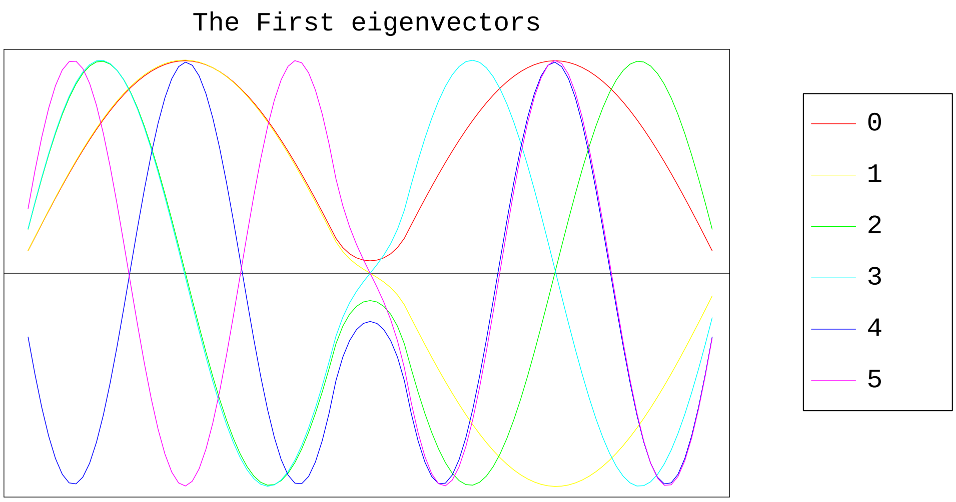

Fig. 4. The first eigenvectors

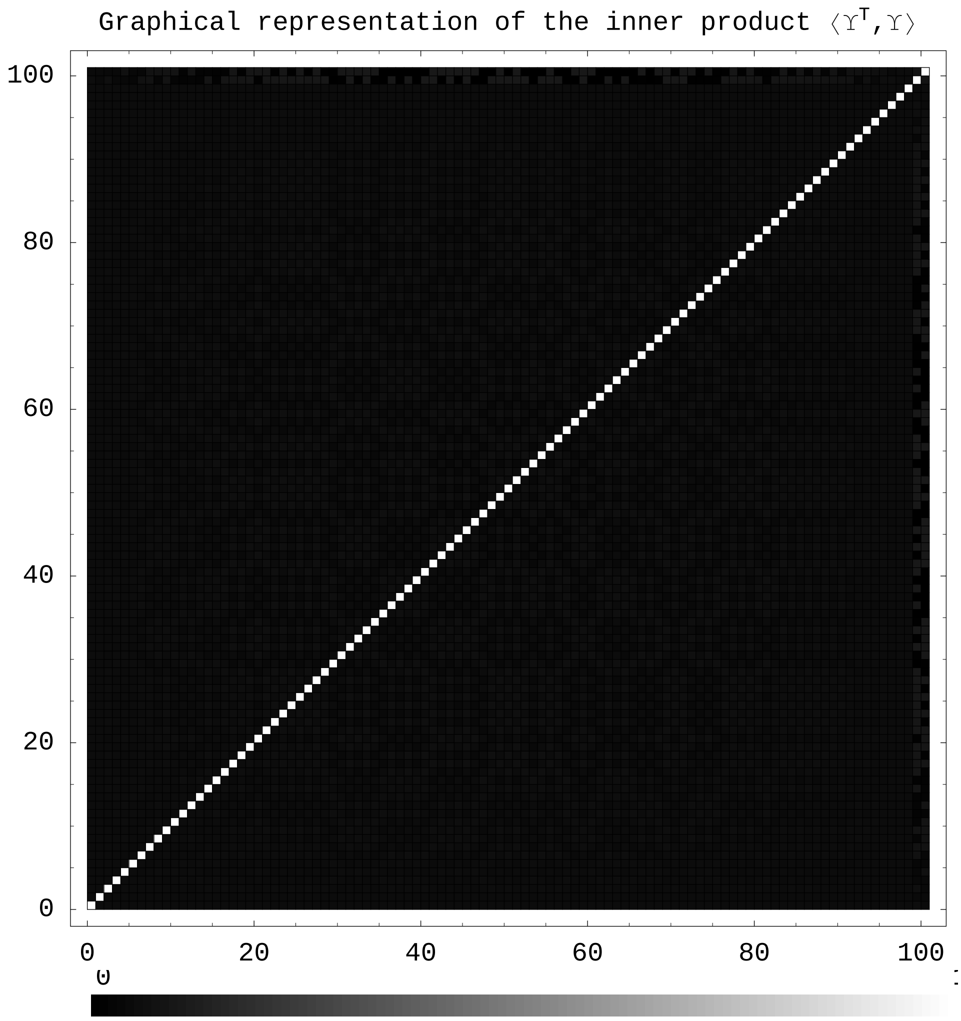

Fig. 5. Graphical representation of the internal product, checking the orthogonality of the eigenvectors ψ_i with respect to the metrics of the *linear* problem.

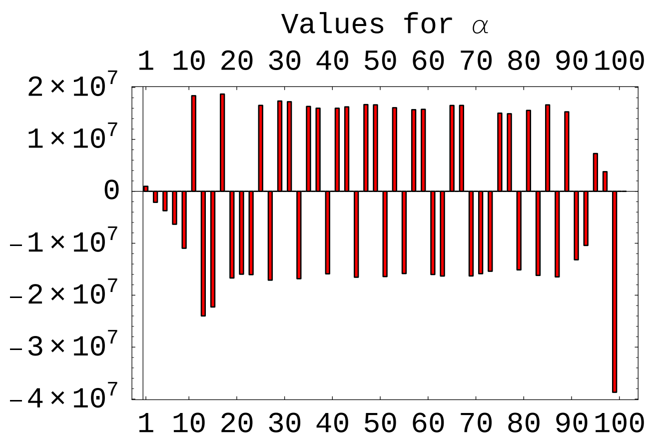

Fig. 6. Coefficient α_i of the projection of the vector $\vec{n}^{(0)}$, on the orthogonal basis of the eigenvectors ψ_j.

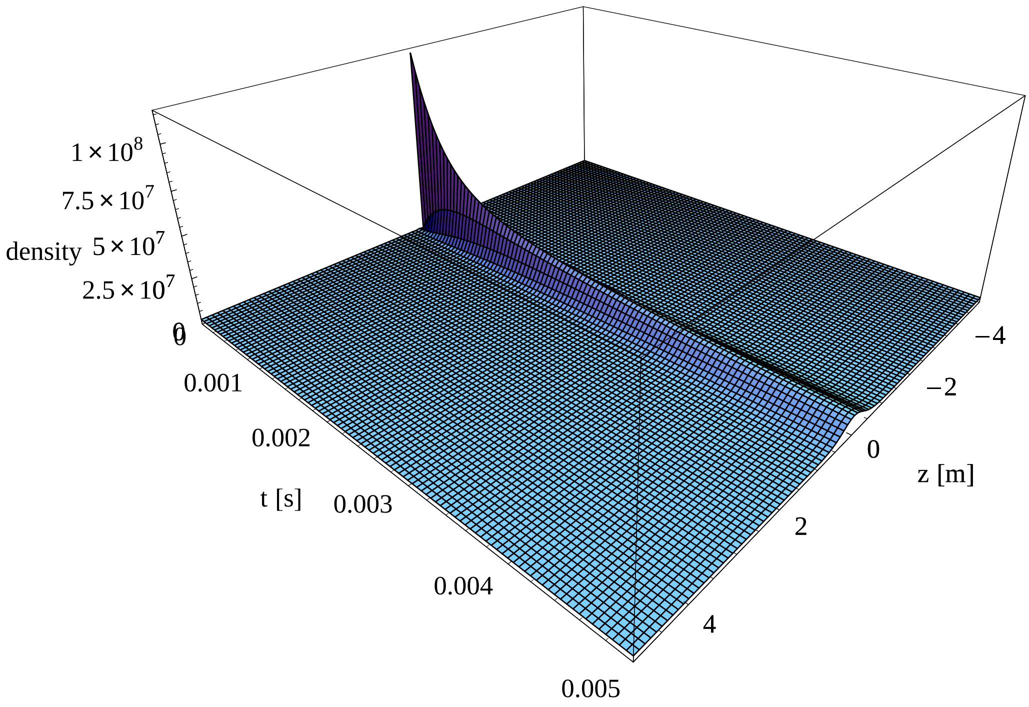

Fig. 7. Eigenvalue problem. Analytical results of the solution eq. (10), in agreement with the numerical solution (Fig. 2). Density profile in a VLHC Stage 2 combined section, beam screen and ion pump sections.

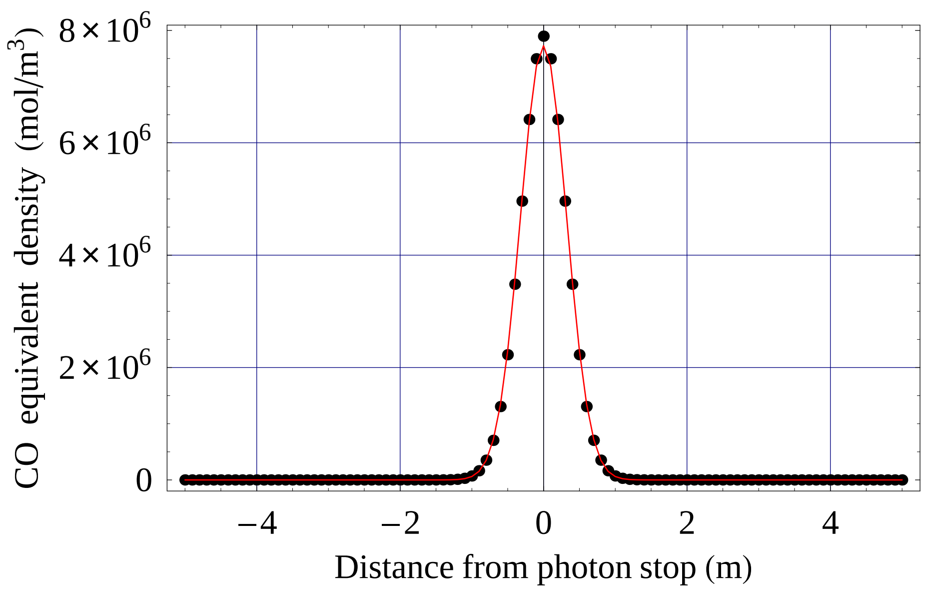

Fig. 8. Computed numerical density (dot) compared with the Green function solution at time $t = 5$ *ms*, for an infinitely long beam screen section.

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